GCE Examinations

Mechanics
Module M2

Advanced Subsidiary / Advanced Level

Paper C

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8 \text{ m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. A particle P of mass 2 kg is subjected to a force **F** such that its displacement, **r** metres, from a fixed origin, O, at time t seconds is given by

$$\mathbf{r} = (3t^2 - 4)\mathbf{i} + (3 - 4t^2)\mathbf{j}.$$

(a) Show that the acceleration of P is constant. **(4 marks)**

(b) Find the magnitude of **F**. **(3 marks)**

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2. A pump raises water from a well 12 metres below the ground and ejects the water through a pipe of diameter 10 cm at a speed of 6 $\mathrm{m\,s^{-1}}$.

Given that the mass of 1 $\mathrm{m^3}$ of water is 1000 kg,

(a) find, in terms of π, the mass of water discharged by the pipe every second, **(4 marks)**

(b) find in kJ, correct to 3 significant figures, the total mechanical energy gained by the water per second. **(4 marks)**

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3. A particle moves in a straight horizontal line such that its velocity, v $\mathrm{m\,s^{-1}}$, at time t seconds is given by $v = 2t^2 - 9t + 4$. Initially, the particle has displacement 9 m from a fixed point O on the line.

(a) Find the initial velocity of the particle. **(1 mark)**

(b) Show that the particle is at rest when $t = 4$ and find the other value of t when it is at rest. **(3 marks)**

(c) Find the displacement of the particle from O when $t = 6$. **(5 marks)**

4.

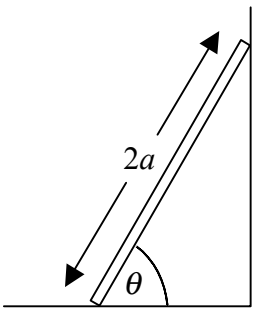

Fig. 1

Figure 1 shows a uniform ladder of mass m and length $2a$ resting against a rough vertical wall with its lower end on rough horizontal ground. The coefficient of friction between the ladder and the wall is $\frac{1}{2}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{3}$.

Given that the ladder is in limiting equilibrium when it is inclined at an angle θ to the horizontal, show that $\tan\theta = \frac{5}{4}$.

(9 marks)

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5. A firework company is testing its new brand of firework, the *Sputnik Special*. One of the company's employees lights a *Sputnik Special* on a large area of horizontal ground and it takes off at a small angle to the vertical. After a flight lasting 8 seconds it lands at a distance of 24 metres from the point where it was launched.

The employee models the firework as a particle and ignores air resistance and any loss of mass which the *Sputnik Special* experiences.

Using this model, find for this flight of the *Sputnik Special*,

(a) the horizontal and vertical components of the initial velocity, **(5 marks)**

(b) the initial speed, correct to 3 significant figures, **(2 marks)**

(c) the maximum height attained. **(3 marks)**

(d) Comment on the suitability of the modelling assumptions made by the employee.

(3 marks)

Turn over

6. Three uniform spheres A, B and C of equal radius have masses $3m$, $2m$ and $2m$ respectively. Initially, the spheres are at rest on a smooth horizontal table with their centres in a straight line and with B between A and C. Sphere A is projected directly towards B with speed u.

Given that the coefficient of restitution between A and B is $\frac{2}{3}$,

(a) show that the speeds of A and B after the collision are $\frac{1}{3}u$ and u respectively.

(6 marks)

The coefficient of restitution between B and C is e. Given that A and B collide again,

(b) show that $e > \frac{1}{3}$. **(8 marks)**

7.

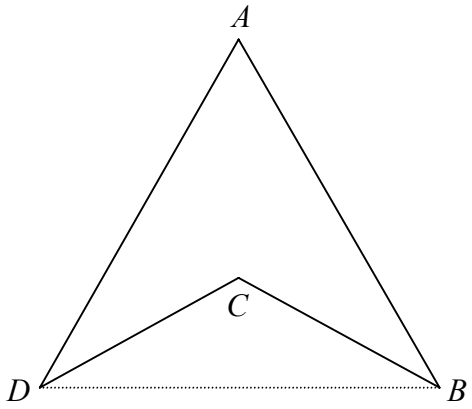

Fig. 2

Figure 2 shows a uniform lamina $ABCD$ formed by removing an isosceles triangle BCD from an equilateral triangle ABD of side $2d$. The point C is the centroid of triangle ABD.

(a) Find the area of triangle BCD in terms of d. **(3 marks)**

(b) Show that the distance of the centre of mass of the lamina from BD is $\frac{4}{9}\sqrt{3}\,d$.

(8 marks)

The lamina is freely suspended from the point B and hangs at rest.

(c) Find in degrees, correct to 1 decimal place, the acute angle that the side AB makes with the vertical.

(4 marks)

END